NONLINEAR MODELS FOR EVOLUTION OF DISTURBANCES ON THE SUPERSONIC BOUNDARY LAYER

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Introduction

Over the last decade the nonlinear interaction of disturbances in supersonic boundary layers has been extensively investigated. It has become possible due to accumulated experience in theoretical and experimental studies of nonlinear disturbance evolution in subsonic flows.

In the case of compressible gas flows, especially at supersonic velocities, the theoretical researches of nonlinear evolution are strongly complicated by the following circumstances. Consideration of temperature and density perturbations raises the order of differential equations, which results in an increase of the calculation volume. At high Mach numbers, additional unstable modes appear. According to the linear theory, the most unstable first mode waves are three-dimensional *(3D)* or oblique, the direction of their fronts does not coincide with the basic flow direction.

For explanation of dynamics of unstable waves observed at introduction to a supersonic boundary layer of inspected disturbances of large intensity, the nonlinear models of interaction of disturbances are considered. In the models of weaklynonlinear interactions, mostly elaborated for subsonic flows, one distinguishes a coupling mechanism in the resonant triads and the combinational interactions of two oscillations, comprising the processes of waves self-action (self-oscillation modes). The possibilities of realization of three-waves interaction with the order $0(\varepsilon^2)$ for supersonic flows are discussed in [1].

In the present work the study of the action in the resonant triads including the stationary modes and also the study of the second non-linear mechanism realization possibility – interaction in pairs - is conducted. Historically, it was just this, and more specifically, the study of the finite amplitude waves self-action effects was the beginning of the near-wall nonlinearity simulation.

The method and results of numerical simulation of development of disturbances in a supersonic boundary layer on a flat plate at high Reynolds numbers are presented in the paper. The solution is built by expansion with respect to the small parameter. The contributions of linear and quadratic terms are taken into account. The resonance interaction of the fundamental *(2D)*mode of linear instability with *(3D)*mode and stationary *(3D)*mode in the parametric region is studied. The results are compared with experimental data under controlled conditions. The results obtained show that the suggested model accounts successfully for experimental results and that the resonance mechanism can play an important role in formation of the spectrum of dominating disturbances in a supersonic boundary layer.

In the present paper, only the interaction of hydrodynamic waves which exponentially vanishes at infinity is considered, i.e., the problems similar to the case of low-speed flows are discussed.

Formulation of the problem

The governing equations for disturbance evolution in a supersonic boundary layer are the well known Navier – Stokes equations of continuity, energy and state.

The flow parameters can be presented as a sum

$$Q(x, y, z, t)=Q_b(x, y, z)+ \varepsilon Z(x, y, z, t)$$

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Abstract

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where $Q_b(x, y, z)$ is the solution to the stationary equations of motion, ε Z is the disturbances (ε >>1).

We consider the evolution of disturbances in supersonic boundary layer over flat plate at high Reynolds numbers Re = $(U_e x/\nu_e)^{1/2}$, where U_e, ν_e – free-stream velocity and kinematic viscosity, x - distance from the leading edge to a reference point.

Therefore parallel flows is a good aproximation for the basic flow.

The disturbances can be described by a system of nonlinear equations which depend on the basic flow parameters. For weak nonlinearity it is possible to restrict the solution by the contribution from linear and quadratic terms. Besides we shall take into account viscosity and termal conductivity only in the linear terms in highest derivative. Then the eight-component vector-function $Z(u, u_y, v, \rho, T, T_y, w, w_y)$ can be presented in the operational notation as a solution of the system of differential equations

$$\mathrm{L}\,\mathrm{Z}=\varepsilon\ \mathrm{M}(q_{ij},q_{kl}). \tag{1}$$

Here u, v, w are velocity perturbations in x, y, z-directions, T, p are perturbations of temperature and pressure, q is a four-component vector, subscripts y, t, x, z mean the transport coefficients (viscosity and heat conductivity).

The solution of (1) can be obtained by using an expansion on the small parameter ε and a two-scale expansion on the x -coordinate, i.e., we introduce «fast» (x_1) and «slow» scales $X=\varepsilon x$. A possibility of introducing two scales is justified by a large difference in phase and amplitude changing rates. Thus,

$$\partial/\partial x=\partial/\partial x_1+\varepsilon\ \partial/\partial X,\ \ \mathrm{Z}=\mathrm{Z}^0+\varepsilon\,\mathrm{Z}^1.$$

In this case Z satisfied following equation and boundary conditions:

$$\mathrm{L}_0\,\mathrm{Z}^0=0,\ z_1{}^0=z_3{}^0=z_5{}^0=z_7{}^0=0\ \text{ at } y=0,\ y=\infty. \tag{2}$$

The zeroth-order solution Z can be taken in the form:

$$\mathrm{Z}^0=Real(\textstyle\sum a_j(X)\mathrm{Z}^{0j}(X, y)\exp(i\Theta_j)). \tag{3}$$

Coefficients a_j, the phases $\Theta_j=\int\alpha_j(x)dx+\beta_j z-\omega_j t$ and Z^{0j} depend on slow variable X. Solution Z^{0j} is governed by equation (A,B,C,D,G) are matrixes which depend on flow parameters):

$$(-i\omega_j\mathrm{A}+i\alpha_j\mathrm{B}+i\beta_j\mathrm{C}+\mathrm{G})\,\mathrm{Z}^{0j}+\mathrm{D}d\,\mathrm{Z}^{0j}/dy=0$$

with boundary conditions (2). Equation (3) subject to homogeneous boundary conditions (2) form eigenvalue problem which provides complex value α_j for given (ω_j, β_j), $Real(\alpha_j)$ being the streamwise wave number and $-Im(\alpha_j)$ being the growth rate of primary wave. In the next order we obtain the following set of coupled equations (mark «asterisk» means complex conjugate):

$$\mathrm{L}^0\,\mathrm{Z}^1=-\textstyle\sum \mathrm{B}\mathrm{Z}^{0j}\exp(i\Theta_j)).(da_j/dX)+\mathrm{M}(z^{0m}{}_k, z^{0n*}{}_p),$$

where vector M represents nonlinear terms. Applying usual solvability conditions (orthogonality to the solution W^{0j} of adjoint problem) yields the set of amplitude equations:

$$da_1/dX=-Im(\alpha_1)a_1+k_1a_2a_3\exp(i\Delta\varphi_1),$$

$$da_2/dX=-Im(\alpha_{2\,2})a_2+k_2a_1a_3{}^*\exp(i\Delta\varphi_2),$$

$$da_3/dX=-Im(\alpha_3)a_3+k_3a_1a_2{}^*\exp(i\Delta\varphi_3),$$

$$k_j = \int (M_j \cdot W^{0j}) dy / \int (Bz^{0j} \cdot W^{0j}) dy,$$

where $\Delta\varphi$ – phases which are small for resonant triplet.

We are interested, first of all, in evolution of the (2D) linear unstable mode with (3D)mode and stationary (3D)mode which satisfy the resonant conditions:

$$\beta_1 = \beta_2 + \beta_3, \ \omega_1 = \omega_2 + \omega_3.$$

For frame of the combinational interactions we come to the following set, describing the spatial behavior of wave amplitudes:

$$da_1/dX = [-Im(\alpha_1) + (E_{1,1,-1}|a_1|^2 + E_{1,2,-2}|a_2|^2)P_1^{-1}]a_1,$$

$$da_2/dX = [-Im(\alpha_2) + (E_{2,2,-2}|a_2|^2 + E_{2,1,-1}|a_1|^2)P_2^{-1}]a_2,$$

$$P_j = \int W^{0j}\, \partial LZ^{0j}/\partial\alpha_j\, dy, \quad j=1,2.$$

Here E are the non-linear coefficients, connecting the secondary harmonics and original waves (3). Let us write down the structure of several of them:

$$E_{1,1,-1} = \int M_j(Z^{vt}{}_{1,-1}, Z^0{}_1)\, W^{0j} dy + \int M_j(Z^{vt}{}_{1,1}, Z^0{}_{-1})\, W^{0j}\, dy,$$

$$E_{1,2,-2} = \int M_j(Z^{vt}{}_{2,-2}, Z^0{}_1)\, W^{0j} dy + \int M_j(Z^{vt}{}_{1,2}, Z^0{}_{-2})\, W^{0j}\, dy + \int M_j(Z^{vt}{}_{1,-2}, Z^0{}_2)\, W^{0j}\, dy.$$

Here Z^{vt} are the secondary waves, emerging by non-linear self- and cross- interactions; $Z^{vt}{}_{1,-1}$ and $Z^{vt}{}_{2,-2}$ are the zero harmonics, describing deformations of the mean flow characteristics; $Z^{vt}{}_{1,1}$ is the overtone, being created by the first wave, its phase is $\Theta = 2\Theta_1$; $Z^{vt}{}_{1,2}$ is the summary secondary wave with the phase $\Theta = \Theta_1 + \Theta_2$; $Z^{vt}{}_{1,-2}$ is the differential secondary wave with the phase $\Theta = \Theta_1 - \Theta_2$.

Numerical results and discussion

In this work the total temperature was kept constant and equal to 310 K, the Prandtl number was Pr = 0.72, specific heat ratio was γ =1.4, the parameters corresponding to the wind experiments of [2].

The nonlinear interaction for resonant wave triads in the supersonic boundary layer on a flat plate were numerically investigated but we will not represent these.

In the present paper the situation arising at introduction to a boundary layer on a slice has exposed to the theoretical analysis at a Mach number M_e=2 of inspected disturbances the large intensity suffices [2]. The authors have called observable dynamics of disturbances as "abnormal". As against earlier considered, she is characterized by a number of features to explain which within the framework of used non-linear models it fails.

At "abnormal" dynamics is established, that *2D* disturbances are most unstable. In an initial spectrum two wave packets on multiple frequencies (f_1 = 10 kHz – subharmonic frequency are arrested and f_2 = 20 kHz – base frequency), the packet dominates on f_1, the maximum of intensity is necessary on *2D* wave. Downstream mainly the preference (*2D*) character of wave spectrums does not change, the strong growth of disturbances on frequency f_1 is watched (almost in 10 times), the amplification of disturbances on a base frequency f_2 is a little bit less, but all the same it is much more linear. It results that the laminar-turbulent transition happens on 20% closer to a leading edge. There is the distortion of a mean stationary velocity *U* crosswise of boundary layer, which width was increased. The phase velocities of disturbances were above linear on 30-40%.

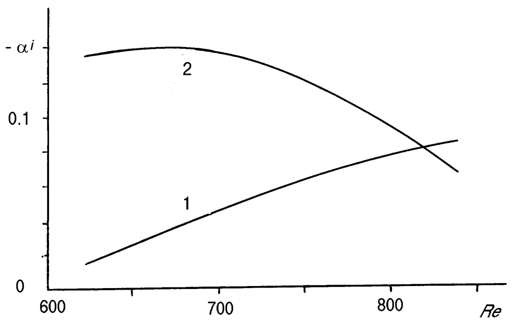


Fig. 1 The linear increments of waves (1 – in f_1. 2 – in f_2).

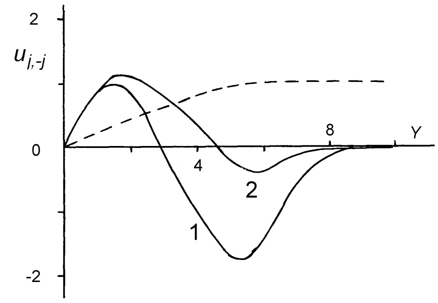


Fig. 2. The secondary zero harmonics $u_{1,-1}$ (1) and $u_{2,-2}$ (2), mean profile U (dash-line).

It was clear that the observable features are called by non-linear character of development of disturbances distinct from three-wave resonant. In the present paper only the combinational interactions of two (*2D*) oscillations of is considered.

In [2] the source of inspected disturbances placed at Re = 497, and the measurements are conducted in range $624 \leq \mathrm{Re} \leq 846$. For these parameters calculations also are conducted. In Fig. 1 we are exhibited increments α_i of linear waves on a subharmonic frequency f_1 and base frequency f_2 (digit 1 and 2). The initial position of a subharmonic at Re = 624 – near to the lower branch of a neutral curve, the linear increment is augmented with growth Re, not reaching a maximum in final cross-section of measurements. The basic wave places near to a maximum of a linear increment, down-stream its increment decreases.

In Fig. 2 we present the secondary harmonicses $u_{1,-1}$ and $u_{2,-2}$ at initial Re on a background of a mean stationary profile U (shaped line). Full deformation U will be recorded $\Delta U=|a_1|^2 u_{1,-1}+|a_2|^2 u_{2,-2}$ also will be determined by amplitudes of incident waves. The registration of nonlinearity is visible that results in the greater property of being filled up of a profile in wall-rigion and occurrence of a defect at an outer boundary, that augments a boundary layer thickness. It completely correlates with experimental observations.

The direction of the influence of non-linear processes of amplitudes primary waves it is possible to analyze considering non-linear factors. Let's unite factors which are in charge of self-effect and for a cross interaction. In Fig. 3 they are exhibited for the researched range Re. The positive values of the non-linear members will call accessory (concerning linear) growth of amplitudes and negative will put to its decrease. From a figure it becomes clear that self-effect of a subharmonic $E_{1,1,-1}$ (full curve 1) will put to increase of amplitude a_1 and on the contrary – self-effect of a wave on f_2 will moderate its amplitude, this process will rise with increase Re (shaped curve 1).

The influence of a base wave on a subharmonic $(E_{1,2,-2})$ consists of increasing amplitude by last at small Re and of decreasing downstream (full curve 2). For a base wave (f_2) availability in a spectrum

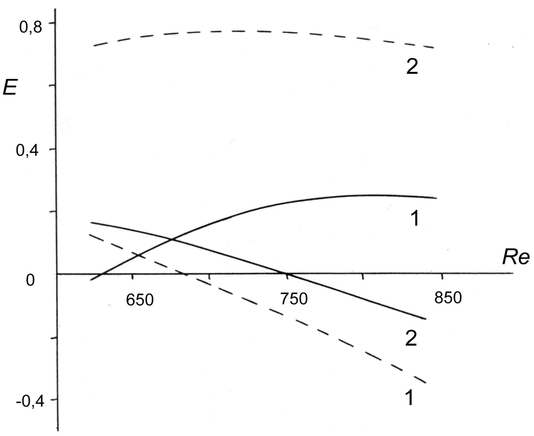
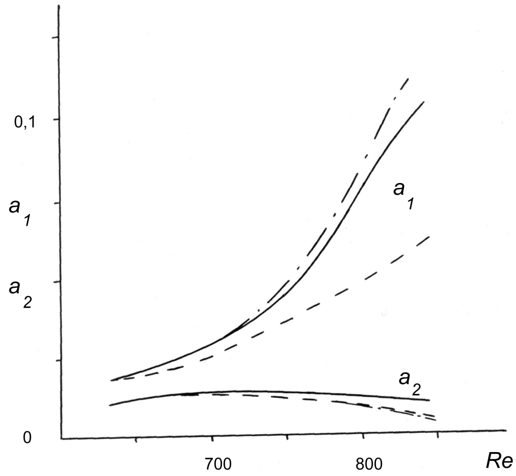


Fig. 3. Non-linear coefficients for the amplitude equations.

of a subharmonic always results in growth a_2 (shaped curve 2).

It is necessary to illustrate considered model on behaviour of amplitudes (Fig. 4). In calculation initial values a_1 and a_2 is corresponded to experimental total intensities of initial wave packets, and $I_1/I_2=3$, and $a_1/a_2=2$.

In figure shaped lines exhibit linear dynamics of amplitudes, appropriate to their linear increments, solid - are shown non-linear amplitudes at combinative process, and by dash-and-dot lines are given non-linear amplitudes in a mode of self-effect.



Rig. 4. The non-linear amplitudes a_1 and a_2.

For a subharmonic the nonlinearity always results in heightened growth of amplitude. Thus a linear degree of growth (relation of final and initial amplitudes) is determined by value ≈ 3 and value – ≈ 6 for non-linear thus at the expense of nonlinearity the amplitude can bring up almost in 2 times. The basic contribution introduces to process self-effect, registration of presence of a wave on basic to frequency lowers a level of accessory amplification because of transfer to her of a part of energy a little.

For this base wave because of considerable reduction linear increments on a researched interval Re the fading linear is watched the amplitudes a_2, nonlinearity in a mode of self-effect practically has not an effect on its value and the influence of a subharmonic results to small to growth of this amplitude. A low-level level of influence of nonlinearity is determined by a smallness of an initial amplitude of this wave.

In non-linear process there is also phase progression of phases which can result in to change of wave numbers $Real(\alpha)$ and phase velocities of disturbances $c=\omega /Real(\alpha)$. In [2] is marked that experimentally particular values of phase velocities $c \approx 0.7$-0.72 on the average on 30-40%exceeded appropriate values of own waves supersonic boundary layer ($c\approx 0.52$-0.55). The phase velocities of waves at non-linear stage are determined. The values of phase velocities with non-linear by the corrections become very close to experimentally particular ($c\approx 0.69$-0.75).

Summarizing said we shall mark that within the framework of the considered combinative model qualitatively prominent feature of dynamics of inspected disturbances of heightened intensity - considerable signal amplification as contrasted to linear, the distortion of mean velocity diagrams correctly are described in the field of outer boundaries resulting in to growth the thickness of the boundary layer and increase of phase velocities of waves.

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REFERENCES

1. **Gaponov S.A., Maslennokova I.**I. Subharmonic instability of a subsonic boundary layer // Thermophysics and Aeromechanics. 1997. Vol. 4, No. 1. P. 1-10.
2. **Kosinov A.D., Ermolaev Y.G., Semionov N.V.** About "abnormal" non-linear wave phenomena in a supersonic boundary layer // Prikl. Mech. Tekh. Fiz. 1999. Vol. 40, No. 5.P. 91-98.